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A Simple Model for Predicting the Flux Distribution
Through the Focal Plane of a
Multifaceted-Concentrator Solar Furnace

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A SIMPLE MODEL FOR PREDICTING THE FLUX-DISTRIBUTION THROUGH THE FOCAL PLANE OF A MULTIFACETED-CONCENTRATOR SOLAR FURNACE

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A model of the flux distribution at the focus of a multifaceted concentrator is presented.

We have described a small doubly-reflecting research furnace.[1] Its optical components, a heliostat and a concentrating array of hexagonal-in-plan-form mirrors, focus sunlight at the aperture of a cavity receiver. A perfect paraboloid of revolution would have given higher concentration ratios. But large paraboloids are difficult to make and manipulate. Small hexagons are convenient and can be close-packed. Spherical mirrors centered on a sphere simplified construction and were economical. In this note we use a simple model to describe the radial distribution of the normal flux at the focus of such an array. Related problems have been considered by others. (See for example [2-5]).

Figure 1 is a schematic representation of a cross-section of the concentrator mirror array in a plane which includes its axis.

In our idealization the sun is a uniform disc which subtends 0.00931 radians. A solar ray bundle reflected from a concentrator

mirror, i, to the focal point of the array, F, produces, in the plane normal to it a circular image of diameter

(1)
$$D_i = 2h_i \sin^2(\phi_i/4) + B_s f$$
,

where ϕ_i is the rim angle of its center, h_i is its effective diameter, B_s is 0.00931 radians, and f is the focal length of the mirror.

In the plane normal to the array axis the solar image from each mirror becomes an ellipse of diminished intensity whose minor axis, $B_i = D_i$, is also given by equation 1. Its major axis is well approximated by

(2)
$$A_i = D_i/\cos\phi_i$$
.

We first construct a flux profile through the plane normal to the array axis assuming perfectly reflecting surfaces. The concentration ratio attributable to a particular concentrator mirror, i, is given by

(3)
$$C_i = A_{ni}/A_{ei} = h_i^2 \cos(\phi_i/2)/4a_ib_i$$

where $A_{n,i}$ is the projected area of mirror i in the plane normal to the array axis and $A_{e,i}$ is the area of the ellipse in the focal plane normal to the array axis, $a_i = A_i/2$, and $b_i = B_i/2 = D_i/2$.

The average concentration ratio to a receiver having an aperture of radius r_j is the sum of the power contributions from each of the mirrors divided by the unconcentrated solar power that would have been delivered to an aperture of equal area. The

smallest aperture which need be considered, $r_{\min}=b_{\min}$, is the minor axis of the ellipse corresponding to the mirror with the smallest rim angle. Within this radius, the flux into the aperture is constant and at its highest value.

In principle, no light falls outside a radius equal to the major axis of the ellipse from a mirror having the largest rim angle. The average concentration ratio depends on the aperture. It is given by

(4)
$$c_{\dot{1}} = P_{\dot{1}}/\pi r_{\dot{1}}^2 I$$

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where I is the solar intensity. With n facets, the power through the aperture is

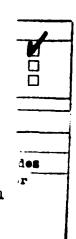
(5)
$$P_j = \sum_{i=1}^n p_{ij}$$
.

The power contributed by each concentrator mirror, p_{ij} , is given by

(6)
$$p_{ij} = F_{ij}IA_icos(0i/2)$$
,

where F_{ij} is the fraction of the ellipse from mirror i captured by the aperture of radius r_j . A_i is the cross-sectional area of a concentrator mirror. For convenience we divided the concentrator mirrors into three sets.

Set 1: $r_j \le b_i$. If the aperture is small, radius r_j is included within the ellipse from mirror i. Some light from mirror i falls outside the aperture. The fraction of the light reflected from mirror i which enters the circle of radius r_j is given by



(7)
$$F_{ij1} = r_j^2/a_ib_i$$
.

Set 2: bi<rj<ai. The aperture fits within the major axis of the ellipse, but it extends over the edges of the minor axis.

Some of the light from mirror i falls outside the aperture, and some of the aperture gets no light from mirror i. In this case

(8)
$$F_{ij2} = 1 - (2/\pi)\sin^{-1}R + (2r_j^2/\pi a_i b_i)\sin^{-1}(b_i R/r_i)$$

where

$$R = ((a_i^2 - r_i^2)/(a_i^2 - b_i^2))^{1/2}$$

Set 3: $r_j \ge a_i$. All the light from mirror i enters the aperture and

(9)
$$F_{ij3} = 1$$
.

The total power into the aperture having the radius $\mathbf{r}_{\hat{\mathbf{j}}}$ is

(10)
$$P_j = \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} IF_{ij}A_i\cos(\phi_i/2)$$
.

Since our concentrator has a six-fold center of symmetry, each rim angle has either a six- or twelve fold degeneracy, substantially simplifying the summation required in equation 10.

An example solution for our furnace is shown in Table 1. The radius of the aperture, r_j is 1.8cm. N is the number of mirrors having the rim angle, ϕ_i , its degeneracy. A_{ni} is the projected area in the plane normal to the furnace axis of the mirrors having the rim angle ϕ_i . This area, multiplied by the solar flux intensity, I, gives the solar power incident on each mirror. The major- and minor-axes of the elliptical images are a_i and b_i . F_{ij}

The next column gives the power supplied to the aperture by a single mirror of rim angle ϕ_i divided by the power incident on the normal area of a mirror; it is the effectiveness with which any particular mirror delivers power to a 3.6cm aperture. The last column gives the concentration ratio furnished by each of the mirror-sets, and its sum is the concentration ratio of the furnace.

Table 2 gives the variation with r; of the average concentration ratio, $C_{\dot{1}}$, in the region bounded by $r_{\dot{1}}$, of an optically ideal furnace having the configuration of our furnace. Table 2 also gives the average concentration ratio, $C(r_i)$, in the annular ring between r_j and the next larger r_j . The variation of $C(r_j)$ with r_i is the flux distribution. The theoretical peak concentration ratio of our furnace is about 12500 suns. The theoretical peak concentration ratio of a perfect parabola of revolution having the same rim angle would, for comparison, be about 23000. Our measured peak concentration ratio was 7100. The difference between theoretical and measured values is attributed to a measured reflectivity of 0.68 which, alone, would have reduced the concentration ratio to 8500, and dispersion due to mirror imperfections and misalignment which was responsible for the balance of the loss.

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TABLE 1. CONTRIBUTIONS OF MIRROR SETS
TO THE POWER DELIVERED TO AN IDEAL FURNACE
OF APERTURE 3.6CM

N	ø i	A _{ni}	$\mathtt{a_{i}}$	$\mathtt{b_i}$	F _{ij}	p_i/IA_i	$\mathtt{NC_i}$
	rad	cm ²	cm	cm			
6	.0780	438.7	1.356	1.351	1	.999	258.6
6		438.0	1.382	1.370	1	.998	258.2
6	.1566		1.396	1.379	1	.997	258.0
12	.2080	436.7	1.437	1.406	1	.995	514.8
6	.2365	436.0	1.466	1.425	1	.993	257.0
6	.2740	434.9	1.510	1.453	1	.991	256.4
12	.2852	434.6	1.524	1.462	1	.990	512.4
6	.3176		1.570	1.491	1	.987	255.5
12	.3471	432.5	1.617	1.520	1	.985	509.2
12	.3656	431.7	1.648	1.539	1	.983	509.0
6		430.3	1.715	1.579	1	.980	253.6
6		429.5	1.749	1.599	1	.978	253.2
12	.4259		1.767	1.610	1	.977	505.9
12	.4494	428.0	1.821	1.640	.9965	.971	502.9
6	.4873		1.915	1.692	.9605	.932	241.2
12	.4946		1.934	1.702	.9512	.922	477.4
12	.5091		1.974	1.723	.9310	.901	466.4
12	.5374	423.3	2.057	1.767	.8860	.854	442.2
6	.5712	421.3	2.165	1.821	.8211	.788	203.9
6	.5779	420.8	2.188	1.833	.8076	.774	200.3
12	.5780	420.8	2.188	1.833	.8074	.734	400.6
12		419.6	2.258	1.867	.7681	.672	380.0
12		417.4	2.382	1.925	.7063	.625	347.6
12	.6555	415.7	2.487	1.972	.6602	.603	323.6
12	.6681	414.8	2.543	1.996	.6380	.592	312.0
6	.6742	414.3	2.570	2.008	.6275	.560	153.2
12	.6929	413.0	2.658	2.045	.5958	.560	290.1
12	.7301	410.1	2.847	2.122	.5361	.501	259.2
6	.7423	409.2	2.914	2.147	.5175	.482 .473	124.8 245.1
12 12	.7482	408.7	2.948	2.160	.5086 .4813	.446	231.0
	.7669 .7791	407.2 406.2	3.057 3.132	2.228	.4641	.446	111.1
6 12	.7971	404.6	3.247	2.269	.4395	.429	209.7
1 2	./7/1	704.0	3.24/	2.209	• 4373	• 403	207.1

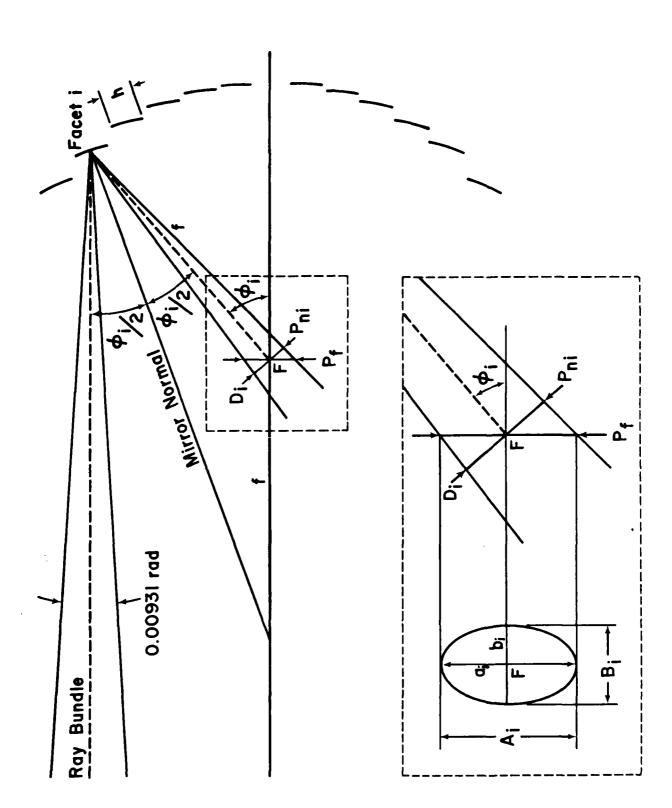
 $c_{j} = 10525.$

TABLE 2. VARIATION OF CONCENTRATION WITH APERTURE

rj	· c _j	C(r _j)
1.0	12534	12532
1.1	12532	12532
1.2	12532	12532
1.3	12532	12151
1.4	12479	10148
1.5	12179	08275
1.6	11706	06728
1.7	11138	05465
1.8	10525	04430
1.9	09900	03254
2.0	09278	02780
2.1	08674	02179
2.2	08097	01590
2.3	07544	01195
2.4	07026	00891
2.5	06545	00633
2.6	06099	00466
2.7	05689	00355
2.8	05315	00243
2.9	04971	00140
3.0	04654	00076
3.1	04364	00034
3.2	04097	00007

FIGURE CAPTIONS

Figure 1. Schematic diagram of a cross-section of the concentrator-mirror array in a plane which includes the axis of the furnace. The inset shows a side view of the focal plane, P_f , and the associated plan view of the elliptical image of the sun produced on the focal plane by one of the mirrors. The dimensions indicated on the figure are defined at the appropriate places in the text.



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